

Electoral Math*

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The following letter was intercepted by our detective service. We publish it here to advise the readers about its contents and significance. Nevertheless, to protect the privacy of the candidates, we have eliminated their true names, the position to which they aspire and the place where the following inquiry was carried out. We recommend the reader to keep his own scores to verify the correctness of the analysis below.

...
...
Market Research and Opinion Surveys
...

...
President
Electoral Institute of ...

Distinguished ...,

According to your very precise wishes we have performed an opinion survey among the inhabitants of ... regarding the four aspiring candidates to the position of... We submit to your consideration the results, summarized in the following table.

*Taken from *Matemáticas electorales* by Luis Mochán published originally in Spanish in the newspaper *La Unión de Morelos*, June 22, 2009, in the weekly column *La Ciencia, desde Morelos para el mundo* in charge of the *Academy of Sciences of the State of Morelos*, inspired on material from the book *A mathematician reads the newspaper* by John Allen Paulos.

Electoral preferences of the inhabitants of...				
<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	First option
<i>D</i>	<i>D</i>	<i>B</i>	<i>C</i>	Second option
<i>C</i>	<i>C</i>	<i>D</i>	<i>B</i>	Third option
<i>B</i>	<i>A</i>	<i>A</i>	<i>A</i>	Fourth option
48%	24%	20%	8%	Percentage

First of all, let me explain the table: Its first row shows that 48% of the electorate favor candidate *A* over candidate *D*, *D* over *C*, and *C* over *B*. Accordingly, the second column shows that 24% prefers *B* over all the others, but in the absence of *B* they would prefer *D*, and, if both were missing, they would favor *C*, as their last choice would be *A*. The other columns are to be read similarly.

Given the previous results, we recommend that

1. You setup a simple election so that *A* wins with 48% of the vote, followed by *B* with 24%, *C* with 20% and finally *D* with 8%.
2. Organize a two-round election. In its first round *C* and *D* would be eliminated. The third column of the table shows that the votes that *C* obtained in the first round would be transferred to *B* in the second round. Analogously, according to the fourth column, as *D* votes in the first round could not be transferred to *C*, having been eliminated, would also be transferred to *B*, which would win with $24\%+20\%+8\%=52\%$ of the vote against 48% for *A*.
3. Arrange an instant runoff voting, in which *D* would be eliminated first after obtaining only 8% of the vote in the first round, which would be transferred to *C*. Then *B*, with only 24% of the vote against 48% for *A* and $20\%+8\%$ for *C*, would be eliminated. According to our table, in the final round between *A* and *C*, *A* would be preferred by only 48% of the electorate, while all of the others would prefer any candidate over *A*. Therefore, *C* would win with 52% of the vote.
4. Organize a weighted election in which each voter would give three points to his favorite candidate, 2 points to his second choice, 1 to his third preference and no point to the remaining candidate. In this case, *D* would win with $(2 \times 48\% + 2 \times 24\% + 1 \times 20\% + 3 \times 8\%) / (3 + 2 + 1 + 0) = (188/6)\% = 31.3\%$ of all the points, while *C* would be second place with $(1 \times 48\% + 1 \times 24\% + 3 \times 20\% + 2 \times 8\%) / 6 = 24.7\%$, *A* third place with $(3 \times 48\% + 0 \times 24\% + 0 \times 20\% + 0 \times 8\%) / 6 = 24\%$, and *B* with last, with $(0 \times 48\% + 3 \times 24\% + 2 \times 20\% + 1 \times 8\%) / 6 = 20\%$.

I take the opportunity to send you my best regards with the sincere hope that the analysis above proves useful to ensure that the candidate of your choice results *democratically* elected.

Yours truly,

(illegible signature)

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