



# Critique of the Post-Election Auditing Recommendations of Verified Voting and the American Statistical Association

## Abstract

Desiring routine accuracy checks of machine-counted U.S. election outcomes, some election advocacy groups have been promoting the post-election auditing recommendations of Verified Voting and The American Statistical Association along with an election auditing bill passed by the New Jersey legislature, S507, as a "model" for post-election auditing.<sup>1</sup>

Unfortunately, the N.J. statute and the recommendations of Verified Voting and The American Statistical Association employ mathematically incorrect assumptions for calculating election audit sample sizes, prohibit the most efficient methods, and include loopholes that would make election audits less effective for detecting outcome-altering vote fraud.

This paper is *not* an exhaustive discussion of all the flaws or benefits of the election auditing recommendations of Verified Voting and the American Statistical Association or the NJ statute.

## 1. The Verified Voting/American Statistical Association/New Jersey Election Auditing Language Allows a Significant Number of Incorrect Election Outcomes

The NJ election audit statute requires auditing:

*“at least 90% (or 99%) statistical power that a 100% manual recount of the voter-verifiable paper records would not alter the electoral outcome reported by the audit.”*

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<sup>1</sup> See: NJ Senate Bill 507 – Mandatory Audit Bill [http://www.njleg.state.nj.us/2006/Bills/S1000/507\\_R1.PDF](http://www.njleg.state.nj.us/2006/Bills/S1000/507_R1.PDF) and *Percentage-Based versus Statistical-Power-Based Vote Tabulation Audits* by John McCarthy, Howard Stanislevic, Mark Lindeman, Arlene S. Ash, Vittorio Addona, and Mary Batchter (September 2007, Revised November 2007) [http://verifiedvoting.org/downloads/TAS\\_paper.pdf](http://verifiedvoting.org/downloads/TAS_paper.pdf) The Brennan Center, a couple of state election advocacy groups, and a recent Ph.D. dissertation by Joseph Hall at UCAL, Berkeley recently promoted these election auditing methods.

The statement that “a 100% manual recount would not alter the outcome” is equivalent to stating that “the election outcome is correct.”<sup>2</sup> Thus, in plain English, the bill’s language can be understood to require that at least 90% (or 99%) of election outcomes (winners) are correct. Two problems with this goal for NJ election audits are:

1. there is no exact way to mathematically translate it to an election audit sample size, and
2. it sets a low standard by allowing up to 10% (or 1%) of all election outcomes (the winners of elections) to be incorrect.

A precise and more stringent goal for election auditing is:

*“If an election outcome is incorrect (in other words, if there is sufficient machine miscount to alter who wins), then at least 90% (or 99% or 95%) of the time, the audit must detect that fact.*

Whereas the NJ statute only requires that 90% (or 99%) of all election outcomes (winners) are correct, the second statement above requires that fewer than 10% (or 1%) of any *incorrect* outcomes would escape detection. Since usually less than 100% of all election outcomes are incorrect prior to an audit, 10% (or 1%) of *incorrect* election outcomes is a far smaller number than 10% (or 1%) of *all* election outcomes. The N.J. auditing legislation by design allows a greater likelihood for certifying incorrect election outcomes than the proposals of most election auditing experts.<sup>3</sup>

The table below shows the weakness of NJ’s auditing language in the case that 100%, 75%, 50%, 25%, 10%, 5% or none of the election outcomes were incorrect prior to auditing. The pink

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<sup>2</sup> Both statements assume that all the paper ballots used in the manual recount were cast by voters and properly secured.

<sup>3</sup> There are a few competent U.S. experts in election auditing mathematics were not invited to collaborate with Verified Voting or the American Statistical Association. One author’s original work was replicated by Verified Voting and the American Statistical Association authors, a habit (of replicating without citing) that one of Verified Voting’s authors had shown since 2005. Verified Voting’s lead author, John McCarthy knowingly declined to cite the original work that his group replicated and touted in *Percentage-Based versus Statistical-Power-Based Vote Tabulation Audits*. The founder of Verified Voting, a Stanford C.S. Professor, told the original author in an email that she should not try to receive credit for her own work for “the good of the country”.

columns show the rate of incorrect election outcomes permitted in NJ. The grey columns show the rate of incorrect outcomes permitted under the more stringent requirement.

%of incorrect election outcomes prior to audit		99%		95%	
		The N.J. Audit	Confidence Audit	The N.J. Audit	Confidence Audit
		%of elections certified with incorrect outcomes		%of elections certified with incorrect outcomes (winners)	
	100%	1.0%	1.0%	5.0%	5.0%
	75%	1.0%	0.8%	5.0%	3.8%
	50%	1.0%	0.5%	5.0%	2.5%
	25%	1.0%	0.3%	5.0%	1.3%
	10%	1.0%	0.1%	5.0%	0.5%
	5%	1.0%	0.1%	5.0%	0.3%
	0%	0.0%	0.0%	0.0%	0.0%

The two proposals are equivalent only when 100% or zero of unofficial reported election outcomes are incorrect prior to auditing. The NJ auditing bill language recommended by Verified Voting and the American Statistical Association thus allows a high-rate of incorrect election outcomes.

The more stringent and mathematically more correct approach requires designing the audit sample size to ensure that fewer than 5% of *incorrect* election outcomes are improperly certified. This results in fewer incorrect certifications because less than 100% of unofficial machine-counted election outcomes (winners) are normally incorrect prior to auditing.

In other words, the following two statements are "not" identical and the second statement is a stronger requirement than the first:

*At most 5% of election outcomes shall be incorrectly certified.*

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If the goal of election auditing is to ensure that the candidates sworn into office are correctly elected by voters, then election audit legislation can require a 90% to 99% “*confidence-level*”, meaning that the maximum chance for an *incorrect* election outcome (winner) to be certified is 100% minus the

confidence-level. For example, if we certify an election outcome at 95% confidence-level then the chance that any *incorrect* outcome is certified is at most 5%.

## **2. Verified Voting and the American Statistical Association Recommend Using an Incorrect Assumption that Would Result in Insufficient Election Audit Samples**

Verified Voting and the American Statistical Association's election auditing recommendations use a mathematically incorrect assumption that results in an election audit sample size that could be insufficient to detect outcome-altering vote miscount. Their audit sample size calculations are:

*“based upon the reasonable assumption that within any election district up to 20% of the total votes cast may have been counted for a candidate or ballot position other than the one intended by the voters.”<sup>4</sup>*

This assumption, a 20% maximum vote count error in all districts, is mathematically incorrect because:

1. in some election districts (precincts) 20% of total votes cast is more than the total error that district (or precinct) could contribute to reversing the election outcome because the contest winner received less than 20% of reported votes cast,<sup>5</sup>
2. using 20% of votes counted, rather than ballots cast ignores the potential for incorrectly recorded undervotes and thus potentially calculates a too-small sample size,
3. actual upper margin error bounds vary from 0% to as high as 200%, much higher than the 40% (2\*20%) maximum assumed by Verified Voting and The American Statistical Association, and
4. there exists a precise, accurate formula for the upper error bounds in each district (or precinct or other auditable vote count).<sup>6</sup>

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<sup>4</sup> Ibid footnote 1.

<sup>5</sup> 20% of votes cast can *not* have been miscounted for the winner within any precinct or other auditable vote count where the winner's reported vote share is less than 20% and in most other cases 20% is much smaller than what is possible because actual within-precinct margin error can be as high as 200%.

<sup>6</sup> The precise formula for upper margin error bound was published by Dopp. The formula was later proven to be mathematically correct in a paper by Philip Stark, and a paper by Aslam, Popa, & Rivest derived the correct formula in an

*The effect of using Verified Voting and the American Statistical Association's N.J. statute approach is to virtually always under-estimate or over-estimate the possibility for error within each auditable district/precinct/vote count. The effect would be to use an insufficient audit sample size, increasing the chances for audits to fail to detect outcome-altering vote fraud.*

The actual upper vote count error bound for each precinct or other auditable vote count depends on the number of ballots cast and the reported votes for each candidate in each. A 200% margin error could occur if the reported vote share were 100% for candidate A and 0% for candidate B, but the true vote shares found in the manual audit were 100% for candidate B and 0% for candidate A.

The *correct* upper bound for the maximum amount of possible margin error (in number of ballots) within any one auditable vote count is the quantity:

$$(b + w - r)$$

where

b = #ballots,

w = #winner's votes, and

r = # runner-up's votes.

Or in vote share (percentage of ballots cast) is equal to  $(1 + w/b - r/b)$ .

The actual margin error between two candidates, A and B, within any vote count is defined as

$$(w_v - r_v) - (w_a - r_a)$$

where

$w_v$  is votes reported for winner

$r_v$  is votes reported for runnerup

$w_a$  is votes counted in audit for winner

$r_a$  is votes counted in audit for runnerup

$$(w_v - r_v) - (w_a - r_a) \leq (b + w - r)$$

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intermediate calculation but then, focused on other issues, unfortunately recommended the same incorrect formula as Verified Voting and The American Statistical Association's authors had previously recommended.

Post-election audits use secured voter-marked paper ballots to determine the actual votes.<sup>7</sup>

Because we assume that a vote fraudster would not raise suspicion by switching *all* available target votes, a correct upper bound for the audit sample size calculations is a constant between zero and one times  $(b + w - r)$ . For example, if we assume that no more than 40% of available margin is shifted, then the upper within precinct error bound would be  $(0.4)(b + w - r)$ .

	(w)	(r)	(u)	(e) =100+(w)-(r)	(1) =0.40*((w)+(r))	(2) =(5)/(e)	=if(2)>(1), Impossible	
Reported Auditable Vote Counts				MARGIN ERROR BOUND between the winner and runner-up			EFFECT OF McCarthy/NJ Error Bounds on Sample Size	
Precinct	Winning candidate share in any vote count	Leading runnerup share in any vote count	Undervotes	ACTUAL	McCarthy/NJ	McCarthy/NJ AS A PERCENTAGE OF ACTUAL (over 100% is impossible)	McCarthy/NJ error bounds are incorrect or inadequate	McCarthy/NJ error bounds make sample size
1	5%	95%		10%	40%	400.0%	-0.36 impossible	too big
2	15%	85%		30%	40%	133.3%	-0.28 impossible	too big
3	25%	65%	10%	60%	36%	60.0%		
4	35%	65%		70%	40%	57.1%		
5	50%	50%		100%	40%	40.0%		
6	55%	45%		110%	40%	36.4%	inadequate	too small
7	65%	35%		130%	40%	30.8%	inadequate	too small
8	75%	10%	15%	165%	34%	20.6%	inadequate	too small
9	85%	15%		170%	40%	23.5%	inadequate	too small
10	95%	5%		190%	40%	21.1%	inadequate	too small

The above chart compares the results of using Verified Voting and the American Statistical Association's incorrect calculations for estimating maximum possible error with the actual upper error bounds for a two candidate race in ten precincts with various vote shares for the winner and loser. The column marked (e) contains the actual upper limit of margin error. Column (1) is the Verified Voting/American Statistical Association assumed margin error (a fixed 40% of votes irrespective of the reported vote shares). Column (2) gives the percentage that the Verified Voting/ASA assumption is of the true upper margin error bound. Notice that in column (2) the Verified Voting/ASA/NJ assumption hits its target 40% of margin error correctly only once when there is a 50% vote share for the winning candidate in an auditable vote count. Otherwise Verified Voting/ASA/NJ's method assumes an amount

<sup>7</sup> Alternatively, audits could use machine-printed, voter-verifiable paper ballot records, although computer scientists have shown that there are at least four ways that machine-printed paper-roll ballot records can be fraudulently manipulated to match erroneous electronic ballot records. UCAL, Santa Barbara Computer Security Group has produced a film showing four ways to fraudulently tamper with machine-printed paper-roll ballot records.

of maximum vote miscount that is either impossibly high or unreasonably low relative to the actual possible error.

Because a winning candidate will more often have higher vote share than the runnerup, the method proposed by Verified Voting and the American Statistical Association for calculating election audit samples will often under-estimate the available margin error and produce too-small election audit sample sizes. (See the right-most column above.)

In other words, following these recommendations of Verified Voting and the American Statistical Association for calculating post-election audit sample sizes would result in insufficient sample sizes to detect outcome-altering vote miscount and would not obtain the advertised “statistical power” that Verified Voting and the American Statistical Association claim.

### **3. Verified Voting/American Statistical Association/New Jersey Post-Election Audits Are Inefficient**

Verified Voting/ASA/NJ election auditing recommendations prohibit using the most efficient methods for calculating audit sample sizes and for randomly selecting vote counts by requiring that “precincts to be audited will be chosen randomly (with equal chance of selection)”. Smaller audit sample sizes can be used if the probability for random selection of each precinct or other auditable vote count is weighted by its upper margin error bound. Weighted random selections may be accomplished using fairly simple arithmetic and ten-sided dice to select the precincts or other auditable vote counts.

Notice that in the pink column in the chart below the Verified Voting/ASA/NJ method for randomly selecting auditable vote counts (precincts or other audit units) selects all auditable vote counts with equal likelihood, so that vote count #1 in the first row, although it could contribute at most only

0.4% (less than half of one percent) of the total margin error to reverse an outcome, has the same chance for selection as vote count #19 that could potentially contribute 16.5% of the total possible margin error.

Comparison of McCarthy/NJ Audit with Confidence Audit - Random Selection Procedures								
Vote Count Identifier	Total Number of Ballots Cast	Winning candidate share in any vote count	Leading runnerup share in any vote count	Undervotes	Maximum available margin error between winner and runnerup	Max Margin Error in Precinct	McCarthy/NJ chance for selecting a vote count	Confidence chance for selecting a vote count weighted by Max Error
1	312	5.0%	90.0%	5.0%	15%	47	5.3%	0.4%
2	306	10.0%	85.0%	5.0%	25%	77	5.3%	0.7%
3	296	15.0%	80.0%	5.0%	35%	104	5.3%	0.9%
4	295	20.0%	75.0%	5.0%	45%	133	5.3%	1.1%
5	293	25.0%	70.0%	5.0%	55%	161	5.3%	1.4%
6	289	30.0%	65.0%	5.0%	65%	188	5.3%	1.6%
7	313	35.0%	60.0%	5.0%	75%	235	5.3%	2.0%
8	319	40.0%	55.0%	5.0%	85%	271	5.3%	2.3%
9	321	45.0%	50.0%	5.0%	95%	305	5.3%	2.6%
10	1070	50.0%	45.0%	5.0%	105%	1,124	5.3%	9.6%
11	1002	55.0%	40.0%	5.0%	115%	1,152	5.3%	9.9%
12	1026	60.0%	35.0%	5.0%	125%	1,283	5.3%	11.0%
13	348	65.0%	30.0%	5.0%	135%	470	5.3%	4.0%
14	350	70.0%	25.0%	5.0%	145%	508	5.3%	4.4%
15	368	75.0%	20.0%	5.0%	155%	570	5.3%	4.9%
16	381	80.0%	15.0%	5.0%	165%	629	5.3%	5.4%
17	387	85.0%	10.0%	5.0%	175%	677	5.3%	5.8%
18	979	90.0%	5.0%	5.0%	185%	1,811	5.3%	15.5%
19	990	95.0%	0.0%	5.0%	195%	1,931	5.3%	16.5%



#### **4. The Verified Voting/American Statistical Association/N.J. Election Audits Include Loopholes that Could Miss Detecting Outcome-Altering Vote Miscount**

New Jersey's bill S507 has a loophole that states that:

*"county and municipal elections held in fewer than 100 election districts are exempt from" being audited... "for county and municipal elections held in 100 or more election districts, any procedure designed, adopted, and implemented by the audit team shall be implemented to ensure with at least 90% statistical power that a 100% manual recount of the voter-verifiable paper records would not alter the electoral outcome reported by the audit."*

This provision permits officials wanting to avoid effective independent audits of election contests by combining voting precincts into large voting centers to put election contests below the 100 precinct threshold. Such requirements have been historically used by some election officials to overcome recount requirements by redefining "precincts" to include a larger number of voters using more than one ballot definition style in each precinct.

#### **5. The N.J. Election Audits Require Using Inefficient Audits Regardless of Future Improvements in Voting System Auditability**

Audits are always more efficient (requires auditing fewer ballots) to achieve the same confidence-level, if the size of the audit unit (auditable vote count) is smaller. However, the NJ Audit statute inflexibly requires auditing "election districts" (analogous to precincts) thus prohibiting the auditing of smaller units, like individual machines or ballots in the case that future voting systems improve to become auditable on these smaller, more efficient levels.

## **6. Verified Voting/American Statistical Association Method for Estimating the Effect of Precinct-Size Variation on Election Audit Sample Size is Inaccurate and Unnecessary**

Verified Voting/American Statistical Association provide an original method for estimating the effect of precinct-size (or other auditable vote count size) variation that is based on Ohio's congressional district #5 2004 election results. Such an estimate may be useful for planning or budgeting purposes, but has limited usefulness, is unnecessary in a real election audit, and would be inaccurate for most applications. Also, VV/ASA did not provide any mathematical derivation to show that their formula is correct - even for the limited case of Ohio's 5<sup>th</sup> district. VV/ASA's method will be inaccurate as the time and place varies because not every election contest will have the same pattern of reported vote count size variation. If VV/ASA method were applied based on patterns in a prior election, such as is currently being proposed for Ohio, then election auditing will be susceptible to reductions in effectiveness over time with changes in the number and relative size of precincts or other auditable units.<sup>8</sup>

The Verified Voting/American Statistical Association estimate is unnecessary during actual election audits because the first step required for any publicly verifiable election audit is to publicly post all the reported auditable vote counts that sum to the unofficial reported totals. Thus it is always possible to use the exact precinct-size variation to calculate election audit sample sizes.

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<sup>8</sup> For instance, Utah election officials have over time artificially inflates the size and thus reduced the number of precincts in order to avoid conducting state-mandated recounts.

## Conclusion

New Jersey's election audit statutes and the recommendations of Verified Voting and The American Statistical Association should *not* be used as a model for other State election auditing bills. Following the recommendations of Verified Voting and the American Statistical Association on election auditing would result in unnecessarily inefficient and ineffective post-election audits.

The same incorrect recommendations of Verified Voting and the American Statistical Association were recently proposed for auditing legislation in Florida, Colorado, and Ohio<sup>9</sup>. In addition, the Brennan Center and others are promoting the same ineffective and inefficient election auditing procedures that would increase the chances that vote fraud would escape detection beyond the advertised claims for such election audits.<sup>10</sup>

On the other hand, if performed competently and scientifically, independent post-election audits not only would provide confidence in U.S. election outcomes but would replace the need for State-mandated recounts in close contests because the scientific audits automatically require a 100% manual count whenever one is needed.

A procedurally and mathematically sound legislative model for conducting scientific post-election audits is *Mandatory Vote Count Audit Proposal for Utah*, posted online at <http://utahcountvotes.org/legislature/ElectionAudits4Utah.pdf>

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<sup>9</sup> The wide proliferation of the incorrect NJ auditing method is in large part due to its promotion by Verified Voting, VoteTrustUSA, The American Statistical Association, and The Brennan Center and the fact that some of these same organizations have deliberately stifled open scientific inquiry and debate on the important topic of election auditing methods. See [http://electionarchive.net/docs\\_pdf/info/US/MathematicianOustedFromAuditConferenceHotel.pdf](http://electionarchive.net/docs_pdf/info/US/MathematicianOustedFromAuditConferenceHotel.pdf)

<sup>10</sup> The Brennan Center and Joseph Hall also failed, as did Verified Voting and the American Statistical Association's auditing groups, to cite the original author's work that was replicated and misattributed. In any field, it is likely to result in a greater number of mathematical errors, incorrect recommendations, and misstatements when persons who make a habit of replicating others' work without fully and correctly citing it. It is a serious concern that such prominent groups unashamedly risk misleading the public in a area that has potential to negatively impact the integrity of the electoral process for years to come.

For more information on this topic see:

*History of Confidence Election Auditing Development & Overview of Election Auditing Fundamentals* <http://electionarchive.org/ucvAnalysis/US/paper-audits/History-of-Election-Auditing-Development.pdf>

*The 2004 Presidential Election - Compendium of Attempts to Dismiss Vote Fraud*  
<http://electionarchive.org/ucvAnalysis/US/IncorrectElectionDataAnalysis-06.pdf>

*Mandatory Vote Count Audit Proposal for Utah*  
<http://utahcountvotes.org/legislature/ElectionAudits4Utah.pdf>

*Mandatory Post-Election Vote Count Audits -- Probability Proportional to Margin Error Bound (PPMEB) Method* (February, 2008) <http://utahcountvotes.org/legislature/VoteCountAudits.pdf>  
or <http://electionmathematics.org/ucvAnalysis/US/paper-audits/VoteCountAudits-PPMEB.pdf>

*Fool Me Once: Checking Vote Count Integrity* (March 3, 2007) Introduces a graphical method for evaluating election auditing proposals.  
<http://electionarchive.org/ucvAnalysis/US/paper-audits/TierElectionAuditEval.pdf>