Tiered Election Audits

Based on Margins between Candidates

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Sufficient Election Audits to Ensure Correct Election Outcomes

For an election audit to detect vote miscount that could wrongly alter an election outcome, the amount of the audit depends on both the margin between leading candidates and the total number of vote counts in a race.

When the margin between candidates is smaller in close elections, a smaller amount of vote miscount can wrongly alter the outcome and so the amount of a manual election audit must be greater.

It is crucial that the number of vote counts manually audited not drop below a minimum number for each particular margin between candidates or the chance to detect miscount becomes very low. Consider a 5% rate of corruption in which one in twenty (20) vote counts is corrupt. One must sample about 20 counts to have around a 70% chance to detect at least one corrupt count.

The ideal way to determine an election audit amount is to use a formula or numerical method that gives a fixed high probability for detecting miscount.[1] However, for simplicity's sake, Larry Norden, Chair of The Brennan Center Task Force on Voting System Security, suggested the idea of a tiered election audit where the margins between leading candidates are divided into ranges and each range is assigned a specific audit percentage that will assure at least 50% chance of detecting outcome-altering vote miscount.

This paper examines a three-tier and a four-tier election audit where each tier is based on a range of margins between candidates and further, it suggests minimum audit amounts for each tier in order to keep the probabilities for detecting corrupt vote counts sufficiently high. A four-tier audit is more effective and efficient than a three-tier audit because it can be structured to achieve a higher minimum certainty of success for all margin ranges for varying amounts of total vote counts, and yet require fewer manual counts for some margins.

The average-sized U.S. House district has 440 precinct vote counts, so probabilities are given for detecting miscount that could alter an election outcome in a race with 440 vote counts.

Table 1: **A Three-Tier Election Audit**

Margins Between Two Leading Candidates	Vote Count Audit Rate	Minimum Number of Vote Counts To Audit (audit no less than this amount or 100% of vote counts, whichever is less)	Certainty for detecting miscount in a race with 440 Vote Counts. Fewer Counts in Race may mean Less Certainty than Listed Here
0.05% to 1%	50%	170	50% to 99%
1%+ to 5%	10%	24	75% to 99%
5% +	2%	6	72% to 99%

[1] See the work of Kathy Dopp and Frank Stenger, and Ronald Rivest. See http://theory.lcs.mit.edu/~rivest/Rivest-OnEstimatingTheSizeOfAStatisticalAudit.pdf and http://electionarchive.org/ucvAnalysis/US/paper-audits/ElectionIntegrityAudit.pdf

Table 2: **A Four-Tier Election Audit**

Margins Between Two Leading Candidates	Vote Count Audit Rate	Minimum Number of Vote Counts To Audit (audit no less than this amount or 100% of vote counts, whichever is less)	Certainty for detecting miscount in a race with 440 Vote Counts. Fewer Counts in Race may mean Less Certainty than Listed Here
0.05% to 0.5%	65%	182	65% to 99%
0.5+% to 2%	15%	44	68% to 97.5%
2% to 5%	5%	14	70% to 95%
5+%	2%	6	72% to 99%

See the "FourTierAudit" spreadsheet
http://electionarchive.org/ucvAnalysis/US/paper-audits/FourTierAudit/FourTierAudit.xls
which anyone may use to try to find a better plan for a tiered audit.[2]

What Are Vote Counts?

Vote Counts may be:

- precinct or polling location counts by precinct op-scan machines, hand counting, or DRE machines, or
- DRE machine counts, or
- batches of ballots counted by hand or by central count optical scan voting systems.

Audits work when all vote counts are of approximately the same number of ballots.[3] Absentee ballots could be counted in batches that are roughly equal to the median size of other vote counts to be audited.[4] Audits are most economical when there are a larger number of smaller-sized vote counts.

Fundamental Requirements for Verifiably Effective Election Audits

In order to effectively assure the integrity of election outcomes, audits must be timely, verifiable, transparent, independent, scientific, and used to correct election results, as well as mathematically sufficient.[5] These measures are required for effective citizen oversight over election audits:

- Commit the data prior to the audit – public release of vote counts for each audited vote count
- Random selections of vote counts and assignment of auditors to count them
- Transparent method of random selection

[2] Row #7 formulas must be manually edited to recalculate minimum audit amounts.
[3] If vote count sizes are very disparate, then a malicious fraudster could hide enough miscount to rig an election in a few of the largest vote counts to avoid detection.
[4] This was suggested by Charlie Strauss. The effect of vote count size variation is discussed in a paper by Jonathan Wand http://wand.stanford.edu/elections/probability.pdf
[5] For audits to be verifiable and transparent, the vote tallies for each candidate and contest in each vote count to be audited and election records must be publicly available prior to the publicly viewable random selection of vote counts; the public must be able to view the manual counts; and the random selection method should be easily understood by the public. Some transparent methods for randomly selecting vote counts are given by Cordero, Wagner, and Dill: http://www.cs.berkeley.edu/~daw/papers/dice-wote06.pdf

- Election records are publicly available that are needed to verify all audit numbers
- Public may view both the random selections and the manual counts
- Actions to uncover the full extent of errors and to correct the election results must be taken when a corrupt vote count is detected, before the election results are made official
- Auditor access to view and investigate security procedures, ballot handling procedures, seals and records necessary to verify ballot integrity

For more information on other important requirements for election audits, see these documents:

Recommendations for Federal Legislation to Ensure the Integrity of Our Democracy
http://electionarchive.org/ucvInfo/US/EI-FederalLegislationProposal.pdf

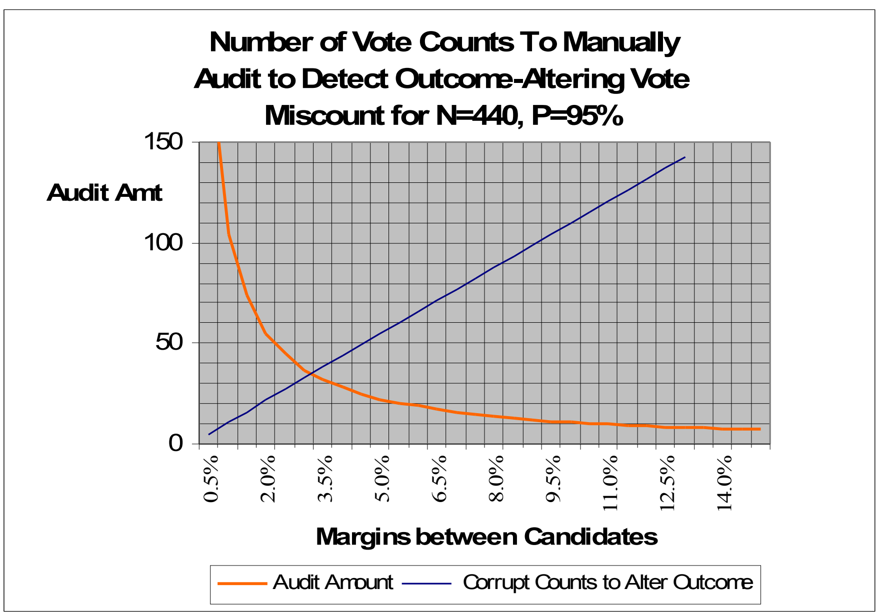
The Election Integrity Audit
http://vote.nist.gov/ElectionIntegrityAudit.pdf

Proposed Mandatory Vote Count Audit Legislation
http://electionarchive.org/ucvAnalysis/US/paper-audits/VoteCountAudit-UT.pdf

The Shape of the Election Audit Curve & Its Effect on Tiered Election Audit Designs

In the chart below, the margin between candidates is along the horizontal axis and the number of vote counts to manually audit is on the vertical axis. As the margin between candidates gets close to zero in a close election (left side of chart), the number of vote counts that must be manually counted to detect outcome-altering vote miscount increases sharply. Therefore any tiered audit design must audit substantially more vote counts when the margin between candidates is less than 1%. The audit curve below begins to flatten more when the margin between candidates is more than 5% (right side of chart).[6]

Chart 1



[6] See "Election Integrity Audit" by Dopp and Stenger for more information and examples of the shape of election audit curves: http://electionarchive.org/ucvAnalysis/US/paper-audits/ElectionIntegrityAudit.pdf

Conclusion

A tiered election audit specifies what percentage and what minimum number of vote counts must be manually counted for specified ranges of margins between candidates obtained from the initial election results.

A tiered election audit is a good compromise between audits which require that “large enough size samples of vote counts are manually counted to ensure 99% scientific certainty that the election outcomes are correct” which must be calculated individually for each race; and audits which require that “2% of all vote counts are manually counted” which are insufficient to ensure the integrity of either close election outcomes or outcomes of races that involve fewer number of total vote counts.[7]

The four-tier audit works better than a three-tier audit because:

- By separating the margins under a half of a percent into a separate category, the audit is more efficient for election officials because sufficient counts for close races can be sufficiently audited without over-auditing when margins are over one half percent.
- Four-tier audits can keep the probability for detecting possible outcome-altering vote miscount higher for all four categories. The probabilities in a four-tier election audit recommended here never falls below a 50% for margins down to 0.05% for any number of total vote counts.
- Four-tier audits can more efficiently with less over-auditing, ensure generally higher probabilities of detecting any outcome-altering miscount.

Tiered election audits are a do-able way to calculate an amount of vote counts to manually audit that would be sufficient to ensure the integrity of U.S. elections.

[7] A tiered election audit is a good compromise between an election audit that requires a fixed high rate of detecting outcome-altering vote miscount in all races, but which would require audit amounts to be calculated individually for each race; and a fixed rate audit that produce very low probabilities of detecting outcome-altering miscount in any close elections whenever the fixed audit rate were less than 30%.

APPENDIX A: The Calculations

The Number of Corrupt Vote Counts to Detect

Assuming that at most 20% of the votes in any one vote count could be wrongly switched to the wrong candidate without raising immediate suspicion[8] then $\frac{\text{margin}}{2*.20}$ is the rate of vote counts that must be corrupt to wrongly alter the outcome. For example, if the margin between candidates is 1%, then $\frac{.01}{.4}=2.5$ % of vote counts must be corrupt to wrongly alter the outcome.

Formula for Calculating Probability of Detecting At Least One Corrupt Vote Count

The formula for calculating the probability of detecting one or more corrupt vote counts in a sample size of S, when C vote counts are corrupt out of N total vote counts is: $P=1-\frac{\binom{C}{0}\binom{N-C}{S}}{\binom{N}{S}}$. This same formula can be implemented in an Excel spreadsheet as: P = 1-HYPGEOMDIST(0,S,C,N)

How to Estimate the Minimum Audit Amount for a Specific Margin

An estimate for the number of vote counts to audit for a specific margin between candidates is: $\frac{-LN(1-P)}{c}$ where P is the desired probability of detecting one or more corrupt vote counts and c is the rate of corrupt counts or $\frac{C}{N}$ where $c=\frac{m}{2v}$ and m is the margin between the leading candidates and v is the maximum rate of vote shift per vote count that is considered to not sufficient to raise immediate suspicion.[9]

One Way to Estimate the Minimum Audit Amount for a Range of Margins

To estimate the minimum audit amount for a range of margins between candidates, because:

- the goal is to preserve a minimum probability of success for the entire range, and
- the shape of the audit curve is steeper where margin values are smaller, and
- the shape of the curve is steepest where the margin is less than 1%,

I averaged the rates of corruption needed to alter an election outcome of the smallest margins in any range of margins to estimate a value for c in the formula above.

For instance, for a three-tier audit with 30%, 10%, and 2% for margins from 0.1% to 1%, 1% to 5%, and 5%+, because the slope of the curve when the margin range is 0.1% to 1% is the steepest, I used the four (4) smallest values of corruption rate needed to alter outcomes of races with margins of 0.1%, 0.2%, 0.3%, and 0.4%. For the margin range of 1%+ to 5% which is not as steeply sloped, I used the five (5) smallest values of corruption rate needed to alter outcomes of races with 1.1%, 1.2%, 1.3%, 1.4%, and 1.5%. I then played around with various values of total vote counts N to see if I could make the probability of success for the entire range fall

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[8] Using this principle, vote counts which exhibit more than a 40% partisan margin shift from prior elections or similar analysis of ratios of outcomes to the partisanship of registered active voters, should be additionally included in any manual audit sample.

[9] See Dopp or Rivest papers for the derivation of this estimate for sample size:
http://electionarchive.org/ucvAnalysis/US/paper-audits/ElectionAuditEstimator.pdf
http://theory.csail.mit.edu/%7Erivest/Rivest-OnEstimatingTheSizeOfAStatisticalAudit.pdf

below 50%. The values I found for the minimum number of vote counts to audit for each range seems to be the minimum value for that range, no matter what the value for total number of vote counts N with the exception of the case when the margin is 0.1% or less and N falls to at or below N=400, in which case, due to the steepness of the audit curve at 0.1%, the probability of success falls below 50% for this particular three-tier audit.

APPENDIX B: EXAMPLE PROBABILITY TABLES

Table 3: **Three Tier Audit for N=400 vote counts** the following chart shows what the resulting probabilities are for detecting at least one outcome-altering vote miscount for election audit rates of 30% , 10%, and 2% with minimum audit amounts of 127, 24, and 6 respectively.

Three Tier Election Audit Evaluation			INPUT	Total Vote Counts =					400
			INPUT	Vote Shift/Count =					20%
			INPUT	Minimum Probability - (used only to calculate a recommended Minimum Audit Amount) =					55%
Margins Ranges			**0.1% to 1%**		**1%+ to 5%**		**5%+ up**		
Audit Rate Choices			**30%**	**20%**	**10%**	**5%**	**3%**	**2%**	**1%**
Floor Audit Amount for Margins			**127**		**24**			**6**	
Actual Audit Amount for N = 400			127	127	40	24	12	8	6
Actual Audit Rate for N = 400			31.8%	31.8%	10.0%	6.0%	3.0%	2.0%	1.5%
Margins Between Candidates Exact	**Corruption Rate That will Alter Outcome**	**#Corrupt Counts to Alter Outcome**	**RESULTS: Probabilities for Detecting At Least One Corrupt Vote Count for Each Margin Between Candidates**						
0.10%	0.25%	1	**31.7%**	31.7%	10.0%	6.0%	3.0%	2.0%	1.5%
0.20%	0.50%	2	**53.5%**	53.5%	19.0%	11.7%	5.9%	4.0%	3.0%
0.30%	0.75%	3	**68.3%**	68.3%	27.2%	17.0%	8.8%	5.9%	4.4%
0.40%	1.00%	4	**78.5%**	78.5%	34.5%	22.0%	11.5%	7.8%	5.9%
0.50%	1.25%	5	**85.4%**	85.4%	41.1%	26.7%	14.2%	9.7%	7.3%
0.60%	1.50%	6	**90.1%**	90.1%	47.1%	31.2%	16.8%	11.5%	8.7%
0.70%	1.75%	7	**93.3%**	93.3%	52.5%	35.4%	19.3%	13.3%	10.1%
0.80%	2.00%	8	**95.4%**	95.4%	57.3%	39.3%	21.8%	15.0%	11.5%
0.90%	2.25%	9	**96.9%**	96.9%	61.6%	43.0%	24.2%	16.8%	12.8%
1.00%	2.50%	10	**97.9%**	97.9%	65.6%	46.5%	26.5%	18.5%	14.2%
1.10%	2.75%	11	**98.6%**	98.6%	**69.1%**	49.8%	28.8%	20.2%	15.5%
1.20%	3.00%	12	**99.1%**	99.1%	**72.3%**	52.9%	31.0%	21.8%	16.8%
1.30%	3.25%	13	**99.4%**	99.4%	**75.1%**	55.8%	33.1%	23.4%	18.1%
1.40%	3.50%	14	**99.6%**	99.6%	**77.7%**	58.6%	35.2%	25.0%	19.4%
1.50%	3.75%	15	**99.7%**	99.7%	**80.0%**	61.1%	37.2%	26.5%	20.6%
1.60%	4.00%	16	**99.8%**	99.8%	**82.1%**	63.6%	39.2%	28.1%	21.8%
1.70%	4.25%	17	**99.9%**	99.9%	**84.0%**	65.8%	41.1%	29.6%	23.1%
1.80%	4.50%	18	**99.9%**	99.9%	**85.6%**	68.0%	42.9%	31.0%	24.3%
1.90%	4.75%	19	**99.9%**	99.9%	**87.1%**	70.0%	44.7%	32.5%	25.5%
2.00%	5.00%	20	**100.0%**	100.0%	**88.5%**	71.9%	46.4%	33.9%	26.6%
2.25%	5.63%	23	**100.0%**	100.0%	**91.8%**	76.9%	51.4%	38.0%	30.1%
2.50%	6.25%	25	**100.0%**	100.0%	**93.4%**	79.7%	54.4%	40.6%	32.3%
2.75%	6.88%	28	**100.0%**	100.0%	**95.3%**	83.4%	58.7%	44.3%	35.5%
3.00%	7.50%	30	**100.0%**	100.0%	**96.3%**	85.5%	61.3%	46.7%	37.6%
3.25%	8.13%	33	**100.0%**	100.0%	**97.4%**	88.1%	64.9%	50.1%	40.5%
3.50%	8.75%	35	**100.0%**	100.0%	**97.9%**	89.6%	67.2%	52.3%	42.5%
3.75%	9.38%	38	**100.0%**	100.0%	**98.5%**	91.6%	70.3%	55.3%	45.3%
4.00%	10.00%	40	**100.0%**	100.0%	**98.8%**	92.6%	72.3%	57.3%	47.1%
4.25%	10.63%	43	**100.0%**	100.0%	**99.2%**	94.0%	75.0%	60.1%	49.7%
4.50%	11.25%	45	**100.0%**	100.0%	**99.4%**	94.8%	76.6%	61.9%	51.4%
4.75%	11.88%	48	**100.0%**	100.0%	**99.5%**	95.8%	78.9%	64.4%	53.8%
5.00%	12.50%	50	**100.0%**	100.0%	**99.6%**	96.3%	80.3%	66.0%	55.4%
5.1%	12.75%	51	**100.0%**	100.0%	99.7%	96.6%	81.0%	**66.8%**	56.1%
5.2%	13.00%	52	**100.0%**	100.0%	99.7%	96.8%	81.7%	**67.5%**	56.9%
5.3%	13.25%	53	**100.0%**	100.0%	99.8%	97.0%	82.3%	**68.3%**	57.6%
5.4%	13.50%	54	**100.0%**	100.0%	99.8%	97.2%	82.9%	**69.0%**	58.4%
5.5%	13.75%	55	**100.0%**	100.0%	99.8%	97.4%	83.5%	**69.7%**	59.1%
6.0%	15.00%	60	**100.0%**	100.0%	99.9%	98.2%	86.2%	**73.1%**	62.5%
6.5%	16.25%	65	**100.0%**	100.0%	99.9%	98.8%	88.5%	**76.1%**	65.7%
7.0%	17.50%	70	**100.0%**	100.0%	100.0%	99.2%	90.4%	**78.9%**	68.7%
7.5%	18.75%	75	**100.0%**	100.0%	100.0%	99.4%	92.0%	**81.3%**	71.5%
8.0%	20.00%	80	**100.0%**	100.0%	100.0%	99.6%	93.4%	**83.5%**	74.0%
8.5%	21.25%	85	**100.0%**	100.0%	100.0%	99.7%	94.6%	**85.5%**	76.4%
9.0%	22.50%	90	**100.0%**	100.0%	100.0%	99.8%	95.5%	**87.3%**	78.6%
9.5%	23.75%	95	**100.0%**	100.0%	100.0%	99.9%	96.3%	**88.8%**	80.6%
10.0%	25.00%	100	**100.0%**	100.0%	100.0%	99.9%	97.0%	**90.2%**	82.4%
10.5%	26.25%	105	**100.0%**	100.0%	100.0%	99.9%	97.6%	**91.5%**	84.1%
11.0%	27.50%	110	**100.0%**	100.0%	100.0%	100.0%	98.0%	**92.6%**	85.7%
11.5%	28.75%	115	**100.0%**	100.0%	100.0%	100.0%	98.4%	**93.5%**	87.1%
12.0%	30.00%	120	**100.0%**	100.0%	100.0%	100.0%	98.7%	**94.4%**	88.4%
12.5%	31.25%	125	**100.0%**	100.0%	100.0%	100.0%	99.0%	**95.2%**	89.6%
13.0%	32.50%	130	**100.0%**	100.0%	100.0%	100.0%	99.2%	**95.8%**	90.7%
13.5%	33.75%	135	**100.0%**	100.0%	100.0%	100.0%	99.3%	**96.4%**	91.7%
14.0%	35.00%	140	**100.0%**	100.0%	100.0%	100.0%	99.5%	**96.9%**	92.6%
14.5%	36.25%	145	**100.0%**	100.0%	100.0%	100.0%	99.6%	**97.4%**	93.4%
15.0%	37.50%	150	**100.0%**	100.0%	100.0%	100.0%	99.7%	**97.8%**	94.2%

Table 4: **Four Tier Audit for N=440 vote counts** the following chart shows what the resulting probabilities are for detecting at least one outcome-altering vote miscount for election audit rates of 65% , 15%, 5%, and 2% with minimum audit amounts of 182, 44, 14, and 6 respectively.

Four Tier Election Audit Evaluation									
			INPUT	Total Vote Counts =					440
			INPUT	Vote Shift/Count =					20%
			INPUT	Minimum Probability - (used only to estimate a recommended Minimum Audit Amount) =					55%
Margins Ranges			0.05% to 0.5%	0.5+% to 2.0%		2+% to 5%		5%+ up	
Audit Rate Choices			**65%**	**15%**	**10%**	**5%**	**3%**	**2%**	**1%**
Floor Audit Amount for Margins			**182**	**44**	**26**	**14**	**8**	**6**	**4**
Actual Audit Amount for N = 440			286	66	44	22	14	9	5
Actual Audit Rate for N = 440			65.0%	15.0%	10.0%	5.0%	3.2%	2.0%	1.1%
Margins Between Candidates Exact	Corruption Rate That will Alter Outcome	#Corrupt Counts to Alter Outcome	RESULTS: Probabilities for Detecting At Least One Corrupt Vote Count for Each Margin Between Candidates						
0.05%	0.13%	1	**65.0%**	15.0%	10.0%	5.0%	3.2%	2.0%	1.1%
0.10%	0.25%	2	**87.8%**	27.8%	19.0%	9.8%	6.3%	4.1%	2.3%
0.15%	0.38%	2	**87.8%**	27.8%	19.0%	9.8%	6.3%	4.1%	2.3%
0.20%	0.50%	3	**95.8%**	38.7%	27.2%	14.3%	9.3%	6.0%	3.4%
0.25%	0.63%	3	**95.8%**	38.7%	27.2%	14.3%	9.3%	6.0%	3.4%
0.30%	0.75%	4	**98.5%**	47.9%	34.5%	18.6%	12.2%	8.0%	4.5%
0.35%	0.88%	4	**98.5%**	47.9%	34.5%	18.6%	12.2%	8.0%	4.5%
0.40%	1.00%	5	**99.5%**	55.8%	41.1%	22.7%	15.0%	9.9%	5.6%
0.45%	1.13%	5	**99.5%**	55.8%	41.1%	22.7%	15.0%	9.9%	5.6%
0.50%	1.25%	6	**99.8%**	62.5%	47.1%	26.6%	17.7%	11.7%	6.7%
0.55%	1.38%	7	99.9%	**68.2%**	52.4%	30.3%	20.4%	13.6%	7.7%
0.60%	1.50%	7	99.9%	**68.2%**	52.4%	30.3%	20.4%	13.6%	7.7%
0.70%	1.75%	8	100.0%	**73.1%**	57.3%	33.9%	23.0%	15.4%	8.8%
0.80%	2.00%	9	100.0%	**77.2%**	61.6%	37.2%	25.5%	17.1%	9.9%
0.90%	2.25%	10	100.0%	**80.7%**	65.5%	40.5%	27.9%	18.8%	10.9%
1.00%	2.50%	11	100.0%	**83.6%**	69.1%	43.5%	30.2%	20.5%	11.9%
1.10%	2.75%	13	100.0%	**88.3%**	75.1%	49.2%	34.7%	23.8%	14.0%
1.20%	3.00%	14	100.0%	**90.1%**	77.7%	51.8%	36.8%	25.5%	15.0%
1.30%	3.25%	15	100.0%	**91.6%**	80.0%	54.3%	38.9%	27.0%	16.0%
1.40%	3.50%	16	100.0%	**92.9%**	82.0%	56.6%	40.9%	28.6%	17.0%
1.50%	3.75%	17	100.0%	**94.0%**	83.9%	58.9%	42.9%	30.1%	18.0%
1.75%	4.38%	20	100.0%	**96.4%**	88.4%	65.0%	48.4%	34.5%	20.8%
2.00%	5.00%	22	100.0%	**97.5%**	90.7%	68.6%	51.8%	37.2%	22.7%
2.05%	5.13%	23	100.0%	97.9%	91.7%	**70.2%**	53.4%	38.6%	23.6%
2.10%	5.25%	24	100.0%	98.2%	92.6%	**71.8%**	55.0%	39.9%	24.6%
2.15%	5.38%	24	100.0%	98.2%	92.6%	**71.8%**	55.0%	39.9%	24.6%
2.20%	5.50%	25	100.0%	98.5%	93.4%	**73.3%**	56.5%	41.2%	25.5%
2.25%	5.63%	25	100.0%	98.5%	93.4%	**73.3%**	56.5%	41.2%	25.5%
2.50%	6.25%	28	100.0%	99.1%	95.3%	**77.3%**	60.7%	45.0%	28.1%
2.75%	6.88%	31	100.0%	99.5%	96.6%	**80.8%**	64.6%	48.5%	30.7%
3.00%	7.50%	33	100.0%	99.6%	97.3%	**82.8%**	67.0%	50.8%	32.4%
3.25%	8.13%	36	100.0%	99.8%	98.1%	**85.4%**	70.3%	54.0%	34.9%
3.50%	8.75%	39	100.0%	99.9%	98.7%	**87.7%**	73.3%	57.0%	37.3%
3.75%	9.38%	42	100.0%	99.9%	99.1%	**89.6%**	76.0%	59.8%	39.6%
4.00%	10.00%	44	100.0%	99.9%	99.2%	**90.7%**	77.7%	61.6%	41.1%
4.05%	10.13%	45	100.0%	100.0%	99.3%	**91.2%**	78.4%	62.5%	41.8%
4.10%	10.25%	46	100.0%	100.0%	99.4%	**91.7%**	79.2%	63.3%	42.6%
4.20%	10.50%	47	100.0%	100.0%	99.5%	**92.2%**	79.9%	64.2%	43.3%
4.30%	10.75%	48	100.0%	100.0%	99.5%	**92.6%**	80.7%	65.0%	44.0%
4.40%	11.00%	49	100.0%	100.0%	99.6%	**93.0%**	81.4%	65.8%	44.7%
4.50%	11.25%	50	100.0%	100.0%	99.6%	**93.4%**	82.0%	66.6%	45.5%
4.60%	11.50%	51	100.0%	100.0%	99.7%	**93.8%**	82.7%	67.4%	46.2%
4.70%	11.75%	52	100.0%	100.0%	99.7%	**94.2%**	83.3%	68.1%	46.8%
4.80%	12.00%	53	100.0%	100.0%	99.7%	**94.5%**	83.9%	68.9%	47.5%
4.90%	12.25%	54	100.0%	100.0%	99.8%	**94.8%**	84.5%	69.6%	48.2%
5.00%	12.50%	55	100.0%	100.0%	99.8%	**95.1%**	85.0%	70.3%	48.9%
5.10%	12.75%	57	100.0%	100.0%	99.8%	95.6%	86.1%	**71.7%**	50.2%
5.20%	13.00%	58	100.0%	100.0%	99.9%	95.9%	86.6%	**72.3%**	50.8%
5.30%	13.25%	59	100.0%	100.0%	99.9%	96.1%	87.1%	**73.0%**	51.5%
5.40%	13.50%	60	100.0%	100.0%	99.9%	96.4%	87.6%	**73.6%**	52.1%
5.50%	13.75%	61	100.0%	100.0%	99.9%	96.6%	88.0%	**74.2%**	52.8%
6.50%	16.25%	72	100.0%	100.0%	100.0%	98.2%	92.1%	**80.3%**	59.3%
7.50%	18.75%	83	100.0%	100.0%	100.0%	99.1%	94.9%	**85.1%**	65.0%
8.50%	21.25%	94	100.0%	100.0%	100.0%	99.6%	96.7%	**88.8%**	70.1%
9.50%	23.75%	105	100.0%	100.0%	100.0%	99.8%	97.9%	**91.6%**	74.6%
10.50%	26.25%	116	100.0%	100.0%	100.0%	99.9%	98.7%	**93.8%**	78.5%
11.50%	28.75%	127	100.0%	100.0%	100.0%	100.0%	99.2%	**95.5%**	82.0%