Post-Election Vote Count Audits -- Probability Proportional to Margin Error Bound (PPMEB) Method

How to determine initial audit sample size, make random selections, and perform discrepancy analysis to achieve any desired confidence (say 95% or greater) that any incorrect unofficial election outcome would be corrected before certification.

Executive Summary

Probability Proportional to Margin Error Bound (PPMEB) election auditing methods provide equal confidence in election outcomes using about 30% smaller sample sizes than other methods. The increase in efficiency is due to randomly selecting vote counts for audit based on the amount of possible margin error each vote count could contribute.

The purpose of this presentation is to explain to the lay person, the fundamental concepts and steps for using PPMEB election auditing methods.

Note: These methods were first applied to election auditing by Aslam, Rivest, and Popa. See the Resources slide at the end of this presentation.

First: Fixed (flat) rate vote count audits use a mathematically incorrect approach.

EXAMPLE: A 10% flat rate audit of a race with a reported 1% margin between the winner and runner-up.

In a race with a reported 1% margin, 2 or 3 miscounted vote counts (precincts) in every 100 vote counts could be sufficient to wrongly reverse the winner. Therefore, we would need to manually count approximately 100 vote counts to detect at least one of these 2 or 3 miscounted vote counts per 100.

- If this 1% margin race had taken place in 100 total precincts, a fixed rate 10% audit would require a 10 precinct vote count audit – hopelessly insufficient (far less than the 100 needed) to detect the miscount which could alter its outcome.

- However, if this 1% margin race had taken place in 5,000 total precinct vote counts, a fixed rate 10% audit would require auditing 500 precinct vote count audit – unnecessarily excessive (far more than the 100 needed).

Any fixed rate audit virtually always manually audits too little to detect outcome-altering miscount or more than is necessary.

Unfortunately, this inefficient and ineffective fixed rate approach is used by most states which audit elections today.

Assumption: Vote Count Audits Should Be Designed To Detect Vote Fraud That Could Reverse Race Outcomes

- Pervasive Machine Error is easy to detect with small fixed-rate audits.
- It is more difficult to detect Vote Fraud where miscount could be targeted to a small number of precincts (or vote counts) to reverse an election outcome.
- The closer the contest, the smaller the number of miscounted precincts (or vote counts) that could reverse the outcome.

Closer Races Require Larger Audits

Example 1:

If the margin between the winner and runner-up is 1%, then if only 2% (2 out of every 100) precinct vote counts were sufficiently miscounted it could possibly reverse the outcome and about 100 (1/.01) precincts must be manually counted to detect at least one miscounted precinct.

Example 2:

If the margin between candidates is 50%, then almost all precincts (or vote counts) would need to be miscounted to reverse the outcome and only about 2 (1/.5) precincts must be manually counted to assure that we detect at least one miscounted precinct.

We may assume that vote fraudster would not switch *all* the available target votes. (If zero votes were counted for the runner-up that could raise suspicion.)

- The maximum possible amount of error in the reported margin (in votes) within any vote count which could deny the closest runnerup a rightful win is:

$$\left(r_{i1} - r_{i2} + b_i \right)$$

where $r_{i1} - r_{i2}$ is the reported unofficial vote margin plus b_i the number of ballots cast in vote count (or precinct) i. The maximum possible margin error is 200% if the runnerup has zero reported votes but actually had 100% of voters' votes.

- However, we could assume that if a fraudster altered the margin by more than say 50% of the available margin error within any vote count (or precinct), it would be immediately noticed even without an audit.

[Note: In this author's judgment, if less than 100% of margin error is assumed for the calculations, then a minimum of one vote count should be audited in each jurisdiction in which a race is held in addition to the calculated sample size in order to check for ballot programming errors (ballot definition errors). Assuming a higher limit than 50% is more conservative - gives a larger audit sample size. 100% is the most conservative approach.]

Steps to Determine Sample Size and the Probability for Selecting Each Vote Count

1. Publicly report the unofficial election results.
2. Calculate the margin in ballots between the winner and the closest runner-up.
3. Determine the maximum available margin error within each vote count.
4. Find the sum of maximum available error & calculate a selection probability for each vote count based on its share of available margin error.
5. Determine the number of selection rounds required for the desired confidence (chance to detect outcome-altering miscount).

1. Report the unofficial election results

Standardize the Report Format for Election Results

					State Representative, District #67			State Representative, District #69	
County Jurisdiction	Precinct	Registered Voters	Total Ballots Cast		Walter "Walt" Borla, DEM	Patick L. Painter, REP		Brad King, DEM	
Carbon	02SC 02	72	38		19	15			
Carbon	07EH 07	742	398		277	80			
Carbon	15WS 15	570	335					269	
Carbon	16IH 16	362	165					125	
Carbon	17EP 17	939	482					391	
Carbon	18SP 18	678	396					326	
Carbon	19WN 19	548	318					258	
Carbon	20SU 20	282	180					158	
Carbon	21WH 21	887	533		389	114			
Carbon	22EC 22	626	374					280	
Carbon	25EC 25	470	272					218	

Reporting election results with precinct or other auditable vote count identifiers, total ballots cast and votes counted, with races and candidates listed in the same format and order for all jurisdictions in a state, facilitates quick creation of a state-wide auditable vote count report and quick audit sample size calculation during the canvass period.

2. Calculate the margin in votes between the winner and the closest runner-up

Calculate the Margin M (in reported votes) between the winner & leading runner-up:

$$M = R_1 - R_2$$

M = #votes for winner - #votes for closest runner-up

3. Determine the possible margin error within each vote count.

Calculate the maximum amount of margin error each vote count could contribute between the winner and runner-up

$$e_i = 0.50\left(r_{i1} - r_{i2} + b_i \right)$$

Where e_i is the amount of available margin error in each vote count, and where $r_{i1} - r_{i2}$ is the reported vote margin and b_i the number of ballots in vote count

Caution: This example assumes that a fraudster would avoid suspicion by shifting at most 50% of the available margin within any vote count. However, if such a multiplier of less than one is assumed, then this author recommends that a minimum of one vote count should be selected for auditing in each separate county or township election jurisdiction where a contest is held by selecting one vote count to audit from each county ***after*** the initial random selection. If not, in this author's opinion, then a 100% rather than a 50% multiplier should be used in this calculation. Otherwise an audit might miss a county where ballot programming errors had reversed counts among candidates sufficient to alter an outcome.

4. Find the total possible margin error in all the unofficial vote counts & calculate each vote count's selection probability

Sum the maximum error of all the reported unofficial vote counts

$$E = \sum_i e_i$$

Calculate the weighted selection probability for each vote count by dividing each vote count's margin error by the total error.

$$p_i = \frac{e_i}{E}$$

5. Determine the number of selection rounds needed to give the desired confidence

$$t = \frac{\ln(\alpha)}{\ln(1 - M / E)}$$

Where:

α is 1 minus the confidence (0.05 for a 95% confidence)

M is the margin in number of ballots (from step 2.)

E is the total of possible margin error in all the vote counts (from step 3.)

If M>E, then the error is too small to reverse the outcome, so audit the minimum amount to detect machine error – say one vote count per contest per election jurisdiction.

Note: These methods described on slides 11 to 14, although used in other fields, were first applied to election auditing by Aslam, Popa, & Rivest; and Stark also describes these methods as applied to election audits. See the Resources slide at the end of this presentation.

The Expected Value for Audit Sample Size

The number of selection rounds, t, is the number of times to throw the dice, not the sample size which may be somewhat less.

The expected value, S, for the actual sample size is calculated as:

$$S = \sum_i 1 - (1 - p_i)^t$$

In other words, the expected sample size is therefore the sum of $1-(1-p_i)^t$ for all auditable vote counts (or precincts) in the election contest, given probability p_i for each vote count of being selected in one selection round and t, the number of selection rounds.

The confidence, c, that this method provides to detect outcome-altering miscount is

$$c = 1 - (1 - \frac{M}{E})^t$$

Where:

t = the total number of selection rounds for vote counts or precincts in the contest

M = the margin between the winner and closest runner-up in number of ballots

E = the total possible margin error in all the vote counts or precincts

Note: The confidence is the chance that if there is a set of vote counts or precincts with nonzero margin error for which the total error could wipe out the margin, then we will get at least one vote count in that set in our audit sample .

PPMEB Method			State Senate #10		
Expected Sample Size	6			Winner	D. CHRIS BUTTARS R
PPMEB Method t rounds	6			Race Margin	28.8%
				Calculated Audit Amt (min = #counties)	8
				Audit %	13.3%
				Probability Check	96.3%
				total ballots in race	34,891
				margin in ballots	10,044
				total precincts in race	60
Increase in Efficiency of PPMED				precincts to reverse outcome	19
25.0%				# counties	1
Probablity to Select given 6 rounds	Precinct Selection Probability	Precinct Margin Error Bound	Counties in Race	Ballots Cast in Race	Precincts to Reverse Outcome

The calculations to determine vote count audit sample sizes can be performed in a spreadsheet or using a program.

See Resources Slide in this presentation.

Note: this particular example assumes that a maximum 50% of the available margin error would be miscounted.

Random sampling method – probabilities proportional to error bound (PPEB) for vote count audits

1. Review: Compute the error bounds, e_i, for each machine vote count and the sum of the error bounds, E, where b_i is the number of total ballots, r_{i1} is the vote count for the winner, and r_{i2} is the vote count for the overall runner-up in precinct i. For $1 \leq i \leq n$

$$e_i = r_{i1} - r_{i2} + b_i \qquad E = \sum_{1 \leq i \leq n} e_i$$

2. Compute the selection probabilities p_i for each precinct and the cumulative selection probabilities:

$$p_i = \frac{e_i}{E} \qquad \hat{p}_i = \sum_{1 \leq j \leq i} p_j$$

UNDER CONSTRUCTION_ A weighted vote count audit random sampling method

3. Select the vote counts/precincts where α = 1- confidence. In our example, α = 0.05
   a) Roll four decimal dice to obtain four decimal digits d_1, d_2, d_3, d_4 and combine them to obtain a fraction $x = 0.d_1d_2d_3d_4$ (so that $0 \le x \le 1$)
   b) Determine the unique i such that

$$\hat{p}_{i-1} \le x < \hat{p}_i$$

   f) Mark precinct i for auditing.

4. Manually audit the selected vote counts.

Note: These methods were described by Aslam, Popa, & Rivest (see Resources)

Example: Summary of 95% Confidence PPMEB Audit Amounts for Utah's Nov. 2004 General Election

Inputs	
Confidence-level	95.0%
Assumed upper Limit of Margin Shift per Precinct	50.0%
Count-time in hours per DRE VVPB roll per one race*	0.25
Labor Costs per Man Hour	$14
# Jurisdictions or Counties	29
Average # DRE rolls per precinct/vote count	5
# Persons per counting team	4

Note: This conservative estimate may overestimating slightly the time to count one race on one voter verifiable paper roll and overestimate the number of paper rolls per precinct. Exact audit amounts will depend on the initial random selections and the requirement that at least one vote count is audited from each jurisdiction or county where a race occurs. Counting one race on a DRE paper roll took 15 minutes during Summit County, Utah's November 2006 general election.

PPMEB Summary Data & Cost Estimate	
Total #Precinct w/ 1 Race Audited out of 16240	**807**
Overall Statewide Audit % (of 16240 precinct-races)	**5.0%**
Total # precincts state-wide	1,958
# Supervisors (3 per 29 counties)	87
Hours total to count	1,009
Approx. Man Hours to count	4,035
Approx. Initial Manual Audit counting labor Costs	$57,708
Federal & Statewide Races total precincts audited (overall average 2.1%)	290
State Senate Races total precincts audited (overall average 13.5%)	121
State House Races total precincts audited (overall average 24.2%)	396
Total #Precinct-Races	**16,240**

PPMEB Overall Summary Results	Expected Audit Amt	% of Total Precincts in Race
Federal & Statewide Races		
President & Vice President	52	2.7%
US Senator	29	1.5%
US Rep. District 1	29	1.5%
US Rep. District 2	29	1.5%
US Rep. District 3	30	1.5%
Governor & Lt. Gov	33	1.7%
Atny. General	29	1.5%
Auditor	29	1.5%
Treasurer	30	1.5%
MAX		2.7%
MIN		1.5%

PPMEB Overall Summary Results	Audit Amt	% of Total Precincts in Race
State Senate Races		
State Senate District #1	32	72.7%
State Senate District #6	9	15.0%
State Senate District #8	12	16.4%
State Senate District #10	6	10.0%
State Senate District #13	3	5.3%
State Senate District #14	3	6.1%
State Senate District #16	3	6.5%
State Senate District #19	7	9.6%
State Senate District #20	10	17.2%
State Senate District #23	4	8.3%
State Senate District #24	6	6.9%
State Senate District #25	3	4.9%
State Senate District #27	18	19.4%
State Senate District #29	5	5.8%
MAX		72.7%
MIN		4.9%

Example: Utah's 2004 General Election

These are the 95% confidence Audit Amounts and Percentages for Utah's Federal and State-wide Races and for Utah State Senate Races

Note: Utah has 29 counties. These audit amounts assume that at least one precinct in each county in which each election contest is held is required to be audited.

PPMEB Overall Summary Results	Audit Amt	% of Total Precincts in Race
State House Races		
State Representative District #1	12	33.3%
State Representative District #2	3	11.1%
State Representative District #3	3	13.0%
State Representative District #4	5	21.7%
State Representative District #5	3	13.6%
State Representative District #6	3	12.0%
State Representative District #7	5	23.8%
State Representative District #8	7	30.4%
State Representative District #9	11	61.1%
State Representative District #10	10	38.5%
State Representative District #11	7	25.9%
State Representative District #13	3	10.7%
State Representative District #14	5	19.2%
State Representative District #15	3	11.1%
State Representative District #16	4	14.8%
State Representative District #17	3	12.0%
State Representative District #18	4	13.8%
State Representative District #19	3	10.3%
State Representative District #20	5	17.9%
State Representative District #21	8	32.0%
State Representative District #22	8	40.0%

PPMEB Overall Summary Results	Audit Amt	% of Total Precincts in Race
State House Races		
State Representative District #23	5	27.8%
State Representative District #24	5	17.2%
State Representative District #25	5	14.7%
State Representative District #26	6	35.3%
State Representative District #28	23	76.7%
State Representative District #29	18	94.7%
State Representative District #30	2	7.7%
State Representative District #31	8	30.8%
State Representative District #32	18	100.0%
State Representative District #33	8	36.4%
State Representative District #34	8	34.8%
State Representative District #35	15	65.2%
State Representative District #36	11	33.3%
State Representative District #37	5	15.2%
State Representative District #38	8	44.4%
State Representative District #39	10	47.6%
State Representative District #40	7	24.1%
State Representative District #43	3	12.0%
State Representative District #44	10	37.0%
State Representative District #45	20	74.1%
State Representative District #47	7	30.4%

PPMEB Overall Summary Results	Audit Amt	% of Total Precincts in Race
State House Races		
State Representative District #48	6	22.2%
State Representative District #49	17	63.0%
State Representative District #50	5	19.2%
State Representative District #51	4	15.4%
State Representative District #52	3	12.0%
State Representative District #53	7	17.5%
State Representative District #54	3	6.8%
State Representative District #55	2	5.3%
State Representative District #56	5	19.2%
State Representative District #58	3	17.6%
State Representative District #59	3	17.6%
State Representative District #61	3	13.0%
State Representative District #64	2	10.0%
State Representative District #67	5	16.1%
State Representative District #68	3	7.5%
State Representative District #70	2	5.4%
State Representative District #71	2	7.1%
State Representative District #73	2	3.7%
State Representative District #74	2	5.4%
Max		100.0%
Min		3.7%

Example: Utah's 2004 General Election

95% confidence Precinct Audit Amounts and Percentages for Utah House Representatives for

Note: Utah has 29 counties. The audit amounts assume that at least one precinct in each county in which each election contest is held is required to be audited.

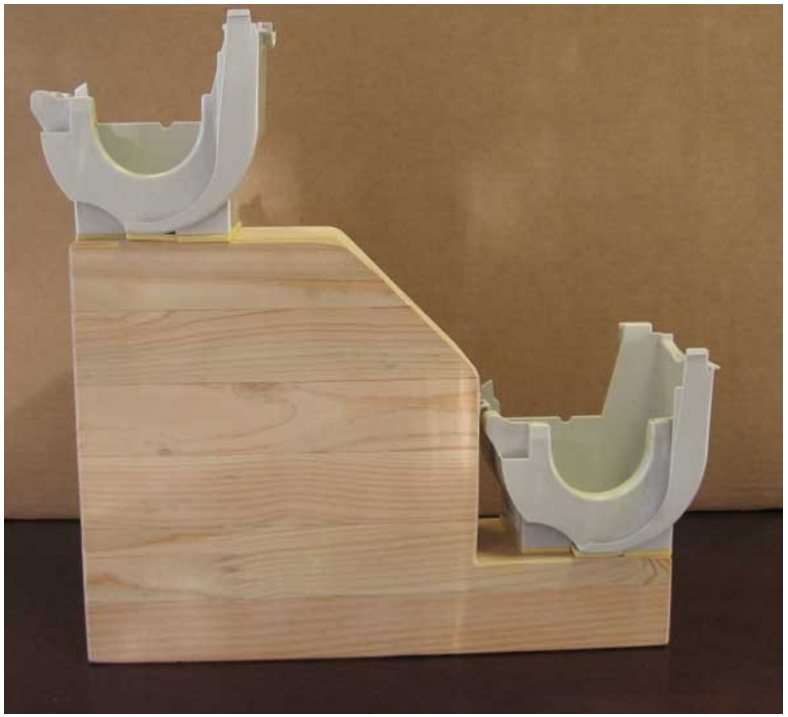
Vote Count Audit Discrepancy – Certify the Election or Expand the Audit?

- ***The methods described in this presentation are incomplete*** because vote count auditors need
  - methods for evaluating the type and amount of discrepancies between the manual audits and the unofficial vote counts in order to determine when to certify the election and when to expand the amount of the manual audit, and
  - procedures for conducting audits that cannot be subverted.
- This presentation will be updated soon to explain methods for evaluating the audit discrepancies and for deciding whether or not to certify the election or to expand the audit.

Boxes used to organize DRE paper roll canisters and extra paper rolls by Salt Lake County, UT

Method used by Salt Lake County, UT to track DRE paper rolls & canisters and ensure that all such materials are returned to the election office.

Sample device for hand-counting DRE paper-rolls used by Salt Lake County, UT



Note: Counting optical scan paper ballots can be done more quickly and accurately with smaller counting teams using a sort and stack method as opposed to using the call and hash mark method which is required for counting DRE paper roll ballot records. Voter marked paper ballots provide a more reliable measure of voter intent as opposed to machine-printed voter-verifiable ballot records.

Resources

- On Auditing Elections When Precincts Have Different Sizes

http://people.csail.mit.edu/rivest/AslamPopaRivest-OnAuditingElectionsWhenPrecinctsHaveDifferentSizes.pdf

- Conservative Statistical Post-Election Audits

http://statistics.berkeley.edu/~stark/Preprints/conservativeElectionAudits07.pdf

- History of the Development of Confidence-level Election Auditing

http://electionarchive.org/ucvAnalysis/US/paper-audits/History-of-Election-Auditing-Development.pdf

- 2-page Summary Handout on Election Auditing

http://utahcountvotes.org/legislature/CriticalAuditMethodology.pdf

- Election audits by sampling with probability proportional to an error bound: dealing with discrepancies by Philip B. Stark, Department of Statistics, U. of CA Berkeley, DRAFT February 2008

http://statistics.berkeley.edu/~stark/Preprints/ppebwrwd08.pdf

- Spreadsheet for calculating audit amounts

http://electionarchive.org/ucvAnalysis/US/paper-audits/UT/GeneralElection2004-Audits-PPMEB.xls

- Mandatory Vote Count Audit (A Legislative Proposal)

http://utahcountvotes.org/legislature/ElectionAudits4Utah.pdf